

Evaluation of Passive Target Tracking Algorithms

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Abstract— The performance evaluation of various standard passive underwater target tracking algorithms like Modified Gain Bearings-only Extended Kalman Filter, Parameterized Modified Gain Bearings-only Extended Kalman Filter and Particle Filter coupled with Modified Gain Bearings-only Extended Kalman Filter using bearings-only measurements is carried out with various scenarios in Monte Carlo Simulation. The performance of Parameterized Modified Gain Bearings-only Extended Kalman Filter is found to be better than all estimates.

Index Terms—Gain, Kalman Filter, Manoeuvring, Performance, Scenario, Sonar, Target tracking,

1 INTRODUCTION

SURVEILLANCE is the most important feature of maritime warfare and is undertaken by active as well as passive sensors. Active methods of surveillance require acoustic transmissions to be made by the surveillance platform and hence susceptible to interception by others. Thus, in certain situations it becomes necessary to maintain silence in active mode.

In the ocean environment, two dimensional bearings-only Target Motion Analysis (TMA) is generally used. **An ownship monitors noisy sonar bearings from a radiating target and finds out Target Motion Parameters (TMP) - viz., range, course, bearing and speed of the target. The basic assumptions are that the target moves at constant velocity most of the time and the ownship motion is unrestricted.** The target and ownship are assumed to be in the same horizontal plane. The problem is inherently nonlinear as the measurement is nonlinear.

The determination of the trajectory of a target solely from bearing measurements is called Bearings-only Tracking (BOT). The BOT area has been widely investigated and numerous solutions for this problem is proposed [1]. In underwater, the ownship can be ship or submarine and the target will be ship, submarine or torpedo. Hence there will be six types of ownship and target scenarios.

In this paper, the required accuracy in the estimated solution is assumed. Hence, the purpose of this paper is performance evaluation of Modified Gain Bearings-only Extended Kalman Filter (MGBEKF), Parameterized Modified Gain Bearings-only Extended Kalman Filter (PMGBEKF) and Particle Filter coupled with Modified Gain Bearings-only Extended Kalman Filter (PFMGBEKF) algorithms with respect to accurate convergence of the solution. The algorithms are evaluated with the scenarios shown in Table 1.

In scenarios 1, 2 and 3 the ownship is assumed to be submarine and target is assumed to be submarine, ship and torpedo respectively. Similarly in scenarios 4, 5 and 6 the ownship is assumed to be ship and target is assumed to be submarine, ship and torpedo respectively. The target range and speeds are chosen as per the scenario. It means that for scenario 1, a submarine and submarine encounter, ownship speed is considered as 3.09 m/s and target speed as 4.12 m/s and initial range as 5 km. In all scenarios the rms error in bearing is assumed to be 0.33°. The algorithms are also evaluated to high bearing error (i.e. lower SNR) of the magnitude of 0.66° rms, as worst condition. In underwater, sometimes outliers in the measurements are inevitable. So it is assumed that 5% of the measurements are with 5 times of the error that is 1.65° (5*0.33°) rms. All these algorithms are evaluated against outliers also.

The algorithms are realised through software and the results in Monte Carlo simulation are presented. A brief discussion of these algorithms is carried out in section 2. The results are presented and the performance evaluation of the algorithms against acceptance criteria is carried out in section 3.

TABLE 1
SCENARIOS CHOSEN FOR EVALUATION OF ALGORITHMS

Scenario	Initial Range (m)	Initial Bearing (deg)	Target Speed (m/s)	Ownship Speed (m/s)	Target Course (deg)
Submarine to Submarine	5000	0	4.12	3.09	135
Submarine to Ship	20000	0	12.36	3.09	135
Submarine to Torpedo	18000	0	20.6	3.09	135
Ship to Ship	20000	0	12.36	12.36	135
Ship to Submarine	5000	0	3.09	12.36	135
Ship to Torpedo	20000	0	20.6	12.36	135

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2 BRIEF DISCUSSION ON PASSIVE TARGET TRACKING ALGORITHMS

2.1 Modified Gain Bearings-only Extended Kalman Filter

The divergence in Extended Kalman Filter (EKF) [2] was eliminated by modifying the gain function and this algorithm is named as Modified Gain Extended Kalman Filter (MGEKF) [3]. The essential idea behind MGEKF is that the nonlinearities be “modifiable”. By eliminating the direct correlation of the gain and measurement noise process in the estimates of MGEKF, the bias in the estimation is avoided. A simplified version of the modified gain function is available in [4]. MGEKF is further modified for underwater applications and the algorithm is named as Modified Gain Bearings-only Extended Kalman Filter (MGBEKF) [5] & [6]. In this paper, MGBEKF is used to compare its performance with those other standard estimators for passive target tracking application.

2.2 Parameterized Modified Gain Bearings-only Extended Kalman Filter

The work presented in [7] is found interesting. The authors of [7] divided the range interval of interest into a number of sub-intervals following geometric progression and each sub-interval was dealt with an independent Kalman filter. They suggested that this method can be extended to course and speed parameterization, if prior knowledge of target course and speed respectively are vague.

In this situation, obtaining fast convergence has an important role and this is achieved using parameterization. Inclusion of range, course and speed, parameterization is proposed for MGBEKF to track a torpedo using bearings-only measurements. This algorithm is named as Parameterized Modified Gain Bearings-Only Extended Kalman Filter (PMGBEKF).

Let the range, course and speed intervals of interest be (maximum-range, minimum-range), (maximum-course, minimum-course) and (maximum-speed, minimum-speed) respectively. The initial weights of each MGBEKF is set to 1/N subsequently, the weight of filter *i* at time *k* is given by

$$\zeta^i(k) = \frac{p(B(k), i) \zeta^i(k-1)}{\sum_{j=1}^N p(B(k), j) \zeta^j(k-1)} \tag{1}$$

Where $p(B(k), i)$ is the likelihood of measurement $B(k)$. Assuming Gaussian statistics, the likelihood $p(B(k), i)$ can be computed as

$$p(B(k), i) = \frac{1}{\sqrt{2\pi\sigma_{inv}^2}} \exp\left[-\frac{1}{2}\left(\frac{B(k) - \hat{B}^i(k, k-1)}{\sigma_{inv}^2}\right)^2\right] \tag{2}$$

Where $\hat{B}^i(k, k-1)$ is the predicted angle at *k* for filter *i* and σ_{inv}^2 is the innovation variance for filter *i* given by

$$\sigma_{inv}^2 = \hat{H}^i(k) P^i(k, k-1) \hat{H}^{iT}(k) + \sigma_B^2 \tag{3}$$

Where $\hat{H}^i(k)$ is the Jacobian of nonlinear measurement function and $P^i(k, k-1)$ is the predicted covariance for filter

i. Let the state estimate of filter *i* be $\hat{X}^i(k, k)$ and its associated covariance be $P^i(k, k)$, then the combined estimate of PMGBEKF is computed using the Gaussian mixture formulas [8] as follows.

$$\hat{X}(k, k) = \sum_{i=1}^N \zeta^i(k) \hat{X}^i(k, k) \tag{4}$$

$$P(k, k) = \sum_{i=1}^N \zeta^i(k) \left[P^i(k, k) + \left(\hat{X}^i(k, k) - \hat{X}(k, k) \right) \left(\hat{X}^i(k, k) - \hat{X}(k, k) \right)^T \right] \tag{5}$$

2.3 Particle Filter coupled with Modified Gain Bearings-only Extended Kalman Filter using Bearings-only measurements

Particle filter is combined with the MGBEKF and the algorithm is named as Particle Filter[12] coupled with Modified Gain Bearings-only Extended Kalman Filter (PFMGBEKF). In this approach, each particle is updated at the measurement time using the MGBEKF and then resampling (if required) is performed using the measurement. This is like running a bank of Kalman filters (one for each particle) initialized with randomly chosen state vectors and then adding a resampling step (if required) after each measurement.

After $X_S(k+1, k)$ is obtained, it can be refined using the MGBEKF measurement-update equations.

$X_S(k+1, k)$ is updated to $X_S(k+1, k+1)$ according to the following MGBEKF equations [9].

$$P(k+1, k)_i = \phi(k+1, k)_i P(k, k)_i \phi^T(k+1, k)_i + \Gamma Q(k+1) \Gamma^T \tag{6}$$

$$G(k+1)_i = P(k+1, k)_i H^T(k+1)_i \left[\sigma_B^2 + H(k+1)_i P(k+1, k)_i H^T(k+1)_i \right]^{-1} \tag{7}$$

$$X_S(k+1, k+1)_i = X_S(k+1, k)_i + G(k+1)_i [B_m(k+1) - h(k+1, X_S(k+1, k)_i)] \tag{8}$$

$$P(k+1, k+1)_i = [I - G(k+1)_i g(B_m(k+1), X_S(k+1, k)_i)] P(k+1, k)_i \times [I - G(k+1)_i g(B_m(k+1), X_S(k+1, k)_i)]^T + \sigma_B^2 G(k+1)_i G^T(k+1)_i \tag{9}$$

Where $G(k+1)$ is Kalman gain, $P(k+1, k)$ is a priori estimation error covariance for the *i*th particle and $g(\cdot)$ is modified gain function $g(\cdot)$ is given by

$$g = \begin{bmatrix} 0 & 0 & \cos B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) & -\sin B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) \end{bmatrix} \tag{10}$$

Since true bearing is not available in practice, it is replaced by the measured bearing to compute the function $g(\cdot)$. In this paper, PFMGBEKF is used to compare its performance with those other standard estimators for passive target tracking application.

3 SIMULATION AND RESULTS

Simulator is developed to create target, ownship and measurements. It is assumed that the ownship is at the origin and bearing is considered with respect to Y-axis, 0-360° and clockwise positive. Target and ownship movements are updated at every second. All one second samples are corrupted by additive zero mean Gaussian noise. It is assumed that the bearing measurements are available continuously at every second. The ownship is assumed to be carrying out S-manoeuver with a turning rate of 1°/s. The ownship moves initially at a course of 90° for a period of 2 min and then it changes to course 270°. At 9th, 16th and 23rd min, the ownship changes its course from 270-90°, 90-270° and 270-90° respectively. The experiment is conducted for 1000 s.

3.1 Initialization of State Vector and its Covariance Matrix

Let the Sonar Range of the Day (SRD) be 20 km that means sonar can detect the ship at the range of maximum 20 km on that particular day. Using this information in MGBEKF, and PFMGBEKF, target state vector position components are initialized with 20 km. As the velocity components of the target are not available, these are each assumed as 10 m/s. (It is known that the submarine target moves at around 3 m/s and torpedo moves at around 17 m/s. As same algorithm is to be used to track ship, submarine and torpedoes, average speed of the underwater vehicles is considered). In PF, it is observed that around 10,000 particles are necessary to obtain good results. When PF is combined with MGBEKF, 1000 particles are sufficient to get the required accuracy in the solution. (It is also seen that by increasing the particles to 10,000 there is no improvement in accuracy of the solution). In PMGBEKF, the range, course and speed sets contain 3-20 km, 0-359° and 3-20 m/s respectively. The elements of range, course and speed sets follow geometric progression. It is assumed that initialized target state vector follows Uniform Density Function. Accordingly the covariance matrix of initial target state vector components is derived for MGBEKF, UKF and PFMGBEKF. In case of MPEKF, it is assumed that the variance of course and speed are 0.5° and 0.1 m/s respectively and the covariance matrix is derived as given in [10].

3.2 Performance Evaluation of the Algorithms

It is assumed that the TMP are said to be converged when the error in the range, course and speed estimates are less than or equal to 10% of the actual range, 5° of the actual course and 20% of the actual speed respectively. As mentioned earlier, in PFMGBEKF-1000 KF's are used. In PMGBEKF range, course and speed sets with 5 elements (in geometric progression) each are used and so 125 KF's work in parallel.

Though it takes more execution time when compared to

other algorithms and less execution time with that of PFMGBEKF, **execution time is not considered to select as right algorithm for passive target tracking as mentioned earlier.** The convergence time to obtain the range, course and speed estimates together with the required accuracies using each algorithm in each scenario is shown in Table 1. **From the results obtained, it is evident PMGBEKF estimates the solution faster when compared to that of other estimators.** For robustness PMGBEKF is tested for the following cases namely -lower SNR and outliers. For the purpose of presentation of the results the bearing error is increased from 0.33 ° to 0.66 ° rms in scenario 1 and the results obtained are shown in Table 3. It is assumed that 5% outliers in underwater do exist and so 5% of the measurements are randomly chosen with 1.65° (5*0.33) rms error. Again Scenario 1 is chosen with outliers and the results obtained are shown in Table 2.

3.3 Detailed Analysis

Scenario 1 is chosen for presentation of the results in detail. The convergence time for range, course and speed estimates with 0.35°rms, 0.66°rms and 5% outliers with 1.65°rms error in bearing measurements is shown in Table 2. The estimates of range, course and speed when the error is 0.33°rms in bearing measurements are plotted with respect to time in Fig. 1., Fig. 2. and Fig. 3. respectively.

TABLE 2
CONVERGENCE TIME OF VARIOUS ALGORITHMS IN SECONDS

Scenario	RMS Error in Bearing, (deg)	MGB EKF	PFMGB EKF	PMGB EKF
1	0.33	461	408	362
	0.66	512	458	430
	5% outliers with 1.65	474	429	385
2	0.33	582	519	385
3	0.33	301	248	280
4	0.33	520	390	380
5	0.33	400	412	300
6	0.33	411	450	360

TABLE 3
CONVERGENCE TIME IN SECONDS FOR RANGE, COURSE AND SPEED ESTIMATES FOR SCENARIO 1

RMS Error in Bearing (deg)	Target parameters	MGB EKF	PMGB EKF	PFMGB EKF
0.33	Range	309	245	300
	Course	461	362	408
	Speed	411	301	400
0.66	Range	399	256	401
	Course	510	430	458
	Speed	430	315	410
5% outliers with 1.65	Range	319	246	360
	Course	474	385	429
	Speed	418	306	403

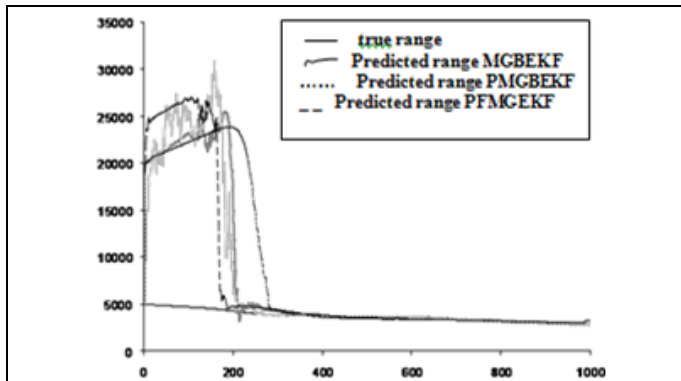


Fig. 1. Estimated Ranges of MGBEKF, PMGBEKF and PFMGBEKF Algorithms

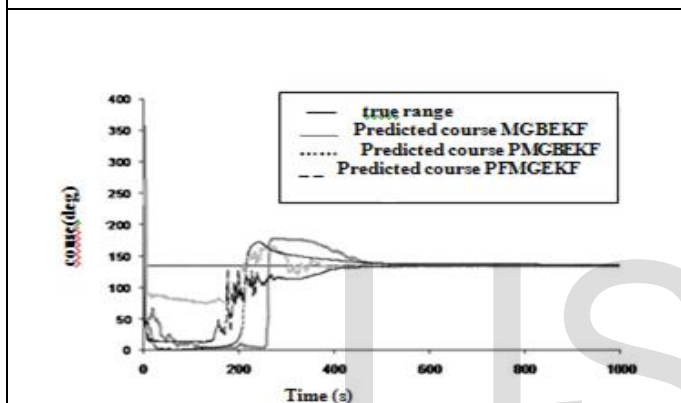


Fig. 2. Estimated Courses of MGBEKF, PMGBEKF and PFMGBEKF Algorithms

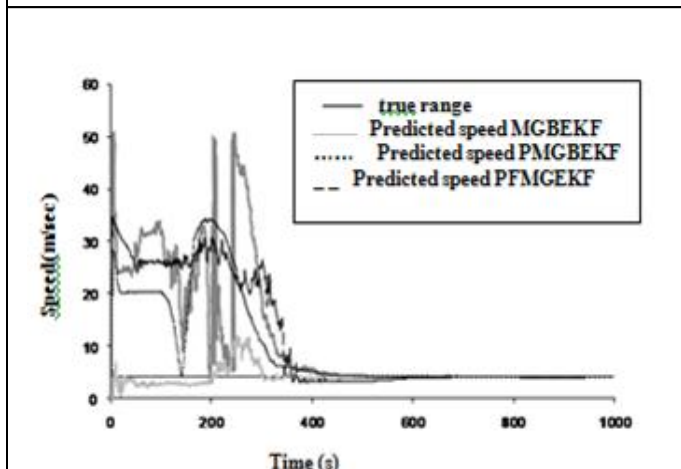


Fig. 3. Estimated Speeds of MGBEKF, PMGBEKF and PFMGBEKF Algorithms

4 CONCLUSION

In underwater, the ownship can be ship or submarine and the target will be submarine, ship or torpedo. Hence there will be six types of ownship and target scenarios as shown in Table 1. Various passive target tracking algorithms shown in Table 2 are considered for comparative study of performance evaluation of algorithms with respect to convergence of the solution. **For robustness, the algorithms are tested against at low SNR and with outliers.** Simulation is carried out and the results are presented in Table 2. **It is observed that PMGBEKF generates the solution faster when compared to that of other estimators.**

From the performance of MGBEKF algorithm it is observed that MGBEKF generates solution faster with few samples when compared to that of other Algorithms. Similar statement is also reported in [11]. It is difficult to say which is better algorithm. But, undoubtedly PMGBEKF generates the solution faster. In PMGBEKF solution converges at around 330 ± 50 s for all types of scenarios because of parameterization in target state vector.

REFERENCES

- [1] Nardone, S. C., Lindgren, A. G., and Gong, K. F., "Fundamental properties and performance of conventional bearings-only target motion analysis", IEEE Trans. Autom. Control, Vol. AC-29, No. 9, Sep. 1984, pp 775-787.
- [2] Aidala, V. J., and Nardone, S. C., "Biased estimation properties of the pseudo linear tracking filter", IEEE Trans. Aerosp. Electron. Syst., Vol. AES-18, No. 4, Jul. 1982, pp 432-441.
- [3] Song, T. L., and Speyer, J. L., "A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearings only measurements", IEEE Trans. Autom. Control, Vol. AC-30, No. 10, Oct. 1985, pp 940-949.
- [4] Galkowski, P. J., and Islam, M. A., "An alternative derivation of the modified gain function of Song and Speyer", IEEE Trans. Autom. Control, Vol. AC-36, No. 11, Nov. 1991, pp 1323-1326.
- [5] Koteswara Rao, S., "Algorithm for detection of manoeuvring targets in bearings-only passive target tracking", IEE Proc. Radar, Sonar & Navigation, Vol. 146, No. 3, Jun. 1999, pp 141-146.
- [6] Koteswara Rao, S., "Modified gain extended Kalman filter with application to bearings-only passive manoeuvring target tracking", IEE Proc. Radar, Sonar & Navigation, Vol. 152, No. 4, Aug. 2005, pp 239-244.
- [7] Ristic, B., Arulampalam, M. S., and Gordon, N., "Beyond Kalman Filters—Particle Filters for Tracking Applications", Artech House, DSTO, 2004.
- [8] Dan Simon, "Optimal State Estimation: Kalman, H ∞ and Nonlinear Approximations", Wiley, 2006.
- [9] Aidala, V. J., and Hammel, S. E., "Utilization of modified polar coordinates for bearings-only tracking", IEEE Trans. Autom. Control, Vol. AC-28, No. 3, Mar. 1983, pp 283-294.
- [10] Koteswara Rao, S., Raja Rajeswari, K., and Linga Murthy, K. S., "Data fusion for underwater target tracking", IET Proc. Radar, Sonar & Navigation, Vol. 4, Iss. 4, 2010, pp 576-585.
- [11] Candy, J. V., "Bayesian Signal Processing: Classical, Modern and Particle Filtering Methods", IEEE and John Wiley & Sons. Inc, 2009.